Majorana neutrino masses, neutrinoless double beta decay, and nuclear matrix elements

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The effective Majorana neutrino mass $m_{\beta\beta}$ is evaluated by using the latest results of neutrino oscillation experiments. The problems of the neutrino mixing pattern, the absolute mass scale of neutrinos, and the effect of *CP* phases are addressed. A connection to the next generation of neutrinoless double beta decay ($0\nu\beta\beta$ decay) experiments is discussed. The calculations are performed for ^{76}Ge, ^{100}Mo, ^{136}Xe, and ^{130}Te by using the advantage of recently evaluated nuclear matrix elements with significantly reduced theoretical uncertainty. The importance of observation of the $0\nu\beta\beta$ decay of several nuclei is stressed.

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I. INTRODUCTION

Strong evidence in favor of neutrino oscillations and small neutrino masses was obtained in the Super-Kamiokande [1], SNO [2], KamLAND [3], and other atmospheric [4,5] and solar [6–9] neutrino experiments. The data from all these experiments are perfectly described by the three-neutrino mixing[1]

$$\nu_{lL}=\sum_{i=1}^{3} U_{li}\nu_{iL}, \quad l=e,\mu,\tau, \tag{1}$$

where ν_i is the field of the neutrino with mass m_i and U_{li} are the elements of the Pontecorvo-Maki-Nakagawa-Sakata unitary neutrino matrix [12]. From the global analysis of the solar and KamLAND data [13] and Super-Kamiokande atmospheric data [1] the following best-fit values of the two independent neutrino mass-squared differences were obtained:

$$\Delta m^2_{\mathrm{sol}}=7.1\times 10^{-5}\ \mathrm{eV}^2,\quad \Delta m^2_{\mathrm{atm}}=2.0\times 10^{-3}\ \mathrm{eV}^2. \tag{2}$$

The observation of neutrino oscillations means that the flavor lepton numbers L_e, L_μ, and L_τ are not conserved by the neutrino mass term. If the total lepton number $L=L_e+L_\mu+L_\tau$ is conserved, neutrinos with definite masses ν_i are Dirac particles. If there are no conserved lepton numbers, ν_i are Majorana particles. The problem of the nature of massive neutrinos (Dirac or Majorana?) is one of the most fundamental ones. The solution of this problem will have very important impact on the understanding of the origin of neutrino masses and mixing.

Investigation of the flavor neutrino oscillations $\nu_l\to\nu_{l'}$ does not allow one to reveal the nature of massive neutrinos ν_i [14,15]. This is possible only via the investigation of the processes in which the total lepton number L is not conserved. The neutrinoless double β decay [16–20]

$$(A,Z)\to(A,Z+2)+e^-+e^- \tag{3}$$

is the most sensitive process to the violation of the total lepton number and small Majorana neutrino masses.

By assuming the dominance of the light neutrino mixing mechanism[2] the inverse value of the $0\nu\beta\beta$-decay half-life for a given isotope (A,Z) is given by [16–20]

$$\frac{1}{T^{0\nu}_{1/2}(A,Z)}=|m_{\beta\beta}|^2|M^{0\nu}(A,Z)|^2 g_A^4 G^{0\nu}_{01}(E_0,Z). \tag{4}$$

Here, $G^{0\nu}(E_0,Z)$, g_A, and $|M^{0\nu}(A,Z)|$ are, respectively, the known phase-space factor (E_0 is the energy release), the effective axial-vector coupling constant, and the nuclear matrix element, which depends on the nuclear structure of the particular isotope under study. The main aim of the experiments on the search for $0\nu\beta\beta$ decay is the measurement of the effective Majorana neutrino mass $m_{\beta\beta}$.

Under the assumption of the mixing of three massive Majorana neutrinos the effective Majorana neutrino mass $m_{\beta\beta}$ takes the form

$$m_{\beta\beta}=U^2_{e1}m_1+U^2_{e2}m_2+U^2_{e3}m_3. \tag{5}$$

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[1]There exist at present indications in favor of $\overline{\nu}_\mu\to\overline{\nu}_e$ transitions, obtained in the accelerator LSND experiment [10]. The LSND data can be explained by neutrino oscillations with $\Delta m^2_{\mathrm{LSND}}\simeq 1\ \mathrm{eV}^2$. The result of the LSND experiment will be checked by the MiniBooNE experiment at Fermilab [11].

[2]Note that there are many other $0\nu\beta\beta$-decay mechanisms triggered by exchange of heavy neutrinos, neutralinos, gluinos, leptoquarks, etc. [16,17,21–25]. However, the observation of the $0\nu\beta\beta$ decay would mean that neutrinos are massive Majorana particles irrespective of the mechanism of this process [26].

The predictions for $m_{\beta\beta}$ can be obtained by using the present data for the oscillation parameters. Its value depends strongly on the type of neutrino mass spectrum and minimal neutrino mass [27–36].

The $0\nu\beta\beta$ decay has not been seen experimentally until now. The best result have been achieved in the Heidelberg-Moscow (HM) ^{76}Ge experiment [37] ($T_{1/2}^{0\nu} \geq 1.9\times 10^{25}$ yr). By assuming the $0\nu\beta\beta$-decay matrix element of Ref. [38] and the result of the HM experiment [37] we end up with the upper limit on the effective Majorana mass $|m_{\beta\beta}| \leq 0.55$ eV. Recently, some authors of the HM Collaboration have claimed the experimental observation of the $0\nu\beta\beta$ decay of ^{76}Ge with half-lifetime $T_{1/2}^{0\nu} = (0.8–18.3)\times 10^{25}$ yr (best-fit value of 1.5×10^{25} yr) [39].[3] This work has attracted a lot of attention from both experimentalists and theoreticians due to important consequences for particle physics and astrophysics [41]. Several researchers of the $\beta\beta$-decay community reexamined and criticized the paper, suggesting a definitely weaker statistical significance of the peak [30,42,43]. In any case the disproof or the confirmation of the claim will come from future experiments. A good candidate for a cross-check of the claimed evidence of $0\nu\beta\beta$ decay of ^{76}Ge is the Cuoricino/CUORE experiment [44] in which $0\nu\beta\beta$ decay of ^{130}Te is investigated.

There are many other ambitious projects in preparation, in particular, CAMEO, CUORE, COBRA, EXO, GEM, GENIUS, MAJORANA, MOON, XMASS, etc. [20,44–46]. In the next generation $0\nu\beta\beta$-decay detectors a few tons of the radioactive $0\nu\beta\beta$-decay material will be used. This is a very big improvement as the current experiments use only a few tens of kilograms for the source. The future double beta decay experiments stand to uncover the fundamental nature of neutrinos (Dirac or Majorana), probe the mass pattern, and perhaps determine the absolute neutrino mass scale and look for possible *CP* violation.

The uncertainty in $m_{\beta\beta}$ is an important issue. The precision of the oscillation parameters is expected to be significantly improved in the future neutrino experiments at the JPARC facility [47], in new reactor neutrino experiments [48], in off-axis neutrino experiments [49], in β-beam experiments [50], and in neutrino factory experiments [51]. The primary concern is the nuclear matrix elements. Clearly, the accuracy of determination of the effective Majorana mass from the measured $0\nu\beta\beta$-decay half-life is mainly determined by our knowledge of the nuclear matrix elements. Reliable nuclear matrix elements are required as they guide future choices of isotopes for the $0\nu\beta\beta$-decay experiments.

In this article the problem of the uncertainty of the $0\nu\beta\beta$-decay matrix elements will be addressed. A further development in the calculation of the $0\nu\beta\beta$-decay ground state transitions will be indicated. By using the latest values of the neutrino oscillation parameters the possible values of the effective Majorana mass $|m_{\beta\beta}|$ will be calculated. By using the nuclear matrix elements of Ref. [38] with reduced theoretical uncertainty, the perspectives of the proposed $0\nu\beta\beta$-decay experiments (CUORE, GEM, GENIUS, Majorana, MOON, EXO, and XMASS) in discerning the normal, inverted, and almost degenerate neutrino mass spectra will be studied.

The paper is organized as follows: In Sec. II the problem of the calculation of the $0\nu\beta\beta$-decay matrix elements will be discussed. The sensitivity of future $0\nu\beta\beta$-decay experiments to the lepton number violating parameter $m_{\beta\beta}$ will be established. In Sec. III the effective Majorana neutrino mass $m_{\beta\beta}$ will be calculated by using the data of neutrino oscillation experiments and assumptions about the character of the neutrino mass spectrum. Conclusions about the discovery potential of planned $0\nu\beta\beta$-decay experiments will be drawn. In Sec. III we present the summary and our final conclusions.

II. UNCERTAINTIES OF THE $0\nu\beta\beta$-DECAY NUCLEAR MATRIX ELEMENTS

A reliable value (limit) for the fundamental particle physics quantity $m_{\beta\beta}$ can be inferred from experimental data only if the nuclear matrix elements governing the $0\nu\beta\beta$-decay are calculated correctly, i.e., the mechanism of nuclear transitions is well understood [17,18].

The nuclear matrix element $M^{0\nu}(A,Z)$ is given as a sum of Fermi, Gamow-Teller, and tensor contributions:

$$M^{0\nu}(A,Z) = -\frac{M_F^{0\nu}(A,Z)}{g_A^2} + M_{GT}^{0\nu}(A,Z) + M_T^{0\nu}(A,Z). \tag{6}$$

The explicit form of the particular matrix elements $M_F^{0\nu}$, $M_{GT}^{0\nu}$, and $M_T^{0\nu}$ can be found in Ref. [52]. In this work, as in most $0\nu\beta\beta$-decay studies [53–58], the higher order terms of the nucleon current were taken into account. Their contributions result in suppression of the nuclear matrix element $M^{0\nu}$ by about 30% for all nuclei. The weak axial coupling constant g_A, which reduces the $M_F^{0\nu}$ contribution to the $0\nu\beta\beta$-decay matrix element, is one of the sources of uncertainty in the determination of $M^{0\nu}$. Usually, it is fixed at $g_A = 1.25$ but a quenched value $g_A = 1.0$ is also considered. The estimated uncertainty of $M^{0\nu}$ due to g_A is of the order of 20% [52].

The evaluation of the nuclear matrix element $M^{0\nu}$ is a complex task for the following reasons:

(i) The nuclear systems that can undergo double beta decay are medium and heavy open-shell nuclei with a complicated nuclear structure. One is forced to introduce many-body approximations in solving this problem.

(ii) The construction of the complete set of states of the intermediate nucleus is needed as the $0\nu\beta\beta$ decay is second order in the weak interaction.

(iii) The confidence level of the nuclear structure parameter choice has to be determined. There are many parameters entering the calculation of nuclear matrix elements, in particular, the mean field parameters, pairing interactions, particle-particle and particle-hole strengths, the size of the

[3]Note that the Moscow participants of the HM Collaboration performed a separate analysis of the data and presented the results [40]. They found no indication in favor of the evidence of $0\nu\beta\beta$ decay.

model space, nuclear deformations, etc. It is required to fix them by a study of associated nuclear processes like single β and $2\nu\beta\beta$ decay, ordinary muon capture, and others. This procedure allows us to assign a level of significance to the calculated $0\nu\beta\beta$-decay matrix elements.

The nuclear wave functions can be tested, e.g., by calculating the two-neutrino double beta decay ($2\nu\beta\beta$ decay)

$$(A,Z)\rightarrow(A,Z+2)+2e^{-}+2\tilde{\nu}_e, \tag{7}$$

for which we have experimental data. The $2\nu\beta\beta$ decay has been directly observed so far in ten nuclides and into one excited state [59]. The inverse half-life of the $2\nu\beta\beta$ decay can be expressed as a product of an accurately known phase-space factor $G^{2\nu}_{01}(E_0,Z)$ and the Gamow-Teller transition matrix element $M^{2\nu}_{GT}(A,Z)$, which is a quantity of second order in the perturbation theory:

$$\frac{1}{T^{2\nu}_{1/2}(A,Z)}=|M^{2\nu}_{GT}(A,Z)|^2 g_A^4 G^{2\nu}_{01}(E_0,Z). \tag{8}$$

The contribution from two successive Fermi transitions is safely neglected as they come from the isospin mixing effect. As $G^{2\nu}_{01}(E_0,Z)$ is free of unknown parameters, the absolute value of the nuclear matrix element $M^{2\nu\, expt}_{GT}(A,Z,g_A)$ can be extracted from the measured $2\nu\beta\beta$-decay half-life for a given g_A.

There are two well established approaches for the calculation of the double beta decay nuclear matrix elements, namely, the shell model [58] and the quasiparticle random phase approximation (QRPA) [17,18]. The two methods differ in the size of the model space and the way the ground state correlations are taken into account. The shell model describes only a small energy window of the lowest states of the intermediate nucleus, but in a precise way. The significant truncation of the model space does not allow one to take into account the β strength from the region of the Gamow-Teller resonance, which might play an important role. Due to the finite model space one is forced to introduce effective operators, a procedure that is not well under control yet [60]. During the period of the last eight years, no progress in shell model calculation of double beta decay transitions has been reported.

The QRPA plays a prominent role in the analysis, in areas inaccessible shell model calculations. It is the most commonly used method for calculation of double beta decay rates [17,18,53–57]. The question is, how accurate is it? For a long period it was considered that the predictive power of the QRPA approach is limited because of the large variation of the relevant $\beta\beta$ matrix elements in the physical window of the particle-particle strength of the nuclear Hamiltonian. Many new extensions of the standard proton-neutron QRPA (*pn*-QRPA), based on the quasiboson approximations, have been proposed.

(i) *The renormalized proton-neutron QRPA* (*pn-RQRPA*) [61,62]. By implementing the Pauli exclusion principle (PEP) in an approximate way in the *pn*-QRPA, one gets the *pn*-RQRPA, which avoids collapse within the physical range of the particle-particle force and offers a more stable solution. The price paid for this is a small violation of the Ikeda sum rule (ISR) which seems to have only a small impact on the calculation of the double beta decay matrix elements. Studies performed within the schematic proton-neutron Lipkin model [63] and realistic calculations of $M^{2\nu}_{GT}$ [17] proved that the RQRPA is a more reliable method than the *pn*-QRPA.

(ii) *The QRPA with proton-neutron pairing [64] and the full RQRPA [56,62]*. The modification of the quasiparticle mean field due to the proton-neutron (pn) pairing interaction affects the single β and $\beta\beta$ transitions. There are some open questions concerning fixing of the strength of the proton-neutron pairing. Recently, it was confirmed within the deformed BCS approach that for nuclei with N much bigger than Z the effect of proton-neutron pairing is small but not negligible [65]. There is the possibility of considering simultaneously both the *pn* pairing and the PEP within the QRPA theory. This version of the QRPA is denoted as the full RQRPA [62] in the literature.

(iii) *The proton-neutron self-consistent RQRPA (pn-SRQRPA) [66]*. The pn-SRQRPA goes a step beyond the *pn*-RQRPA by at the same time minimizing the energy and fixing the number of particles in the correlated ground state instead of the uncorrelated BCS state as is done in other versions of the QRPA. However, the large effect found in the $\beta\beta$ transitions with realistic *NN* interactions [66] is apparently associated with consideration of bare pairing forces, not fitted to the atomic mass differences, within a complicated numerical procedure [67].

(iv) *The deformed QRPA*. Almost all current $\beta\beta$-decay calculations for nuclei of experimental interest were performed by assuming spherical symmetry. Recently, the effect of deformation on the $2\nu\beta\beta$-decay matrix elements was studied within the deformed QRPA. A new suppression mechanism of the $2\nu\beta\beta$-decay matrix elements based on the difference in deformations of the initial and final nuclei was found [68]. It is expected that this effect might be important for the $0\nu\beta\beta$-decay transitions also.

The QRPA many-body approach for description of nuclear transitions is under continuous development. In particular, it has been found feasible to include nonlinear terms in the phonon operator [69]. Another modification of the QRPA phonon operator, which allows exact satisfaction of the ISR, was proposed in Ref. [70]. Thus, further progress in the QRPA calculation of the $\beta\beta$-decay matrix elements is expected.

To estimate the uncertainty of the $0\nu\beta\beta$-decay transition probability, different groups have performed calculations in the framework of different methods (*pn*-QRPA, *pn*-RQRPA, *pn*-SRQRPA, FRQRPA, QRPA with *pn* pairing, and deformed QRPA), different model spaces, and different realistic forces. One might obtain in this way an uncertainty of a factor of 2 to 3, depending especially on the method and the size of the model space. However, significant progress has been achieved in the calculation of the $0\nu\beta\beta$-decay matrix elements recently [38]. It was shown that by fixing the strength of the particle-particle interaction, so that the measured $2\nu\beta\beta$-decay half-life is correctly reproduced, the re-

TABLE I. The current upper limits on the effective Majorana neutrino mass $|m_{\beta\beta}|$ and the sensitivities of the future $0\nu\beta\beta$-decay experiments to this parameter for $A=76$, 100, 130, and 136 nuclei. The $0\nu\beta\beta$-decay matrix elements $M^{0\nu}$ with reduced uncertainty were used [38]. In that calculation the $2\nu\beta\beta$-decay matrix element $M^{2\nu\, expt}_{GT}$ deduced from the half-life $T^{2\nu\, expt}_{1/2}$ was considered. $\mathcal{R}^{2\nu/0\nu}$ is the ratio of the $2\nu\beta\beta$-decay and $0\nu\beta\beta$-decay matrix elements [see Eq. (9)]. $T^{0\nu}_{1/2}$ denotes the current lower limit on the $0\nu\beta\beta$-decay half-life or the sensitivity of planned $0\nu\beta\beta$-decay experiments. The symbols * and † indicate the future sensitivity to $|m_{\beta\beta}|$ of already running and planned $0\nu\beta\beta$-decay experiments, respectively. HM denotes the Heidelberg-Moscow experiment.

Nucleus	$M^{0\nu}$	$M^{2\nu\, expt}_{GT}$ (MeV^{-1})	$\mathcal{R}^{2\nu/0\nu}$ (MeV^{-1})	$T^{2\nu\, expt}_{1/2}$ (yr)	Ref.	$T^{0\nu}_{1/2}$ (yr)	Ref.	Expt.	$\lvert m_{\beta\beta}\rvert$ (eV)
^{76}Ge	2.40	0.15	0.063	1.3×10^{21}	[20]	1.9×10^{25}	[37]	HM	0.55
						3×10^{27}	[20]	Majorana	0.044†
						7×10^{27}	[20]	GEM	0.028†
						1×10^{28}	[20]	GENIUS	0.023†
^{100}Mo	1.16	0.22	0.19	8.0×10^{18}	[20]	6.0×10^{22}	[74]	NEMO3	7.8
						4×10^{24}	[20]	NEMO3	0.92*
						1×10^{27}	[20]	MOON	0.058†
^{130}Te	1.50	0.017	0.013	6.1×10^{20}	[72]	1.4×10^{23}	[72]	CUORE	3.9
						2×10^{26}	[20]	CUORE	0.10*
^{136}Xe	0.98	0.030	0.031	$\geqslant 8.1\times10^{20}$	[20]	1.2×10^{24}	[73]	DAMA	2.3
						3×10^{26}	[20]	XMASS	0.10†
						8×10^{26}	[20]	EXO	0.087†

sulting $M^{0\nu}$ become essentially independent of the considered *NN* potential, the size of the basis, and the restoration of the PEP. The uncertainty of the results obtained for $A=76$, 100, 130, and 136 nuclei has been found to be less than 10%. This an exciting development. It is desired to extend this type of study also to other nuclei and other extensions of the QRPA approach. In this way a correct understanding of the uncertainty of the $0\nu\beta\beta$-decay matrix elements evaluated within the QRPA theory can be established.

The small spread of the $0\nu\beta\beta$-decay results obtained within the procedure of Ref. [38] can be qualitatively understood. It seems that there is an advantage in considering the ratio of the $2\nu\beta\beta$-decay and $0\nu\beta\beta$-decay matrix elements for a given isotope,

$$\mathcal{R}^{2\nu/0\nu}(A,Z)=\left|\frac{M^{2\nu}_{GT}(A,Z)}{M^{0\nu}(A,Z)}\right|, \tag{9}$$

as in this quantity the dependence on the nuclear structure degrees of freedom is suppressed. By assuming

$$M^{2\nu}_{GT}(A,Z)=M^{2\nu\ expt}_{GT}(A,Z,g_A) \tag{10}$$

the absolute value of the $0\nu\beta\beta$-decay matrix element can be inferred. We note that in comparison with $M^{2\nu}_{GT}$, which is evaluated within a nuclear model, the value of $M^{2\nu\, expt}_{GT}$, which is determined from the $2\nu\beta\beta$-decay half-life, depends on g_A. The $2\nu\beta\beta$ decay plays a crucial role in obtaining $0\nu\beta\beta$-decay matrix elements with reduced uncertainty [38].

From the experimental upper limit on the $0\nu\beta\beta$-decay half-life $T^{0\nu\, expt}_{1/2}(A,Z)$, it is straightforward to find a constraint on the effective Majorana neutrino mass $m_{\beta\beta}$ [38]:

$$|m_{\beta\beta}|\leqslant[G^{0\nu}_{01}(E_0,Z)T^{0\nu\ expt}_{1/2}(A,Z)]^{-1/2}\frac{1}{g_A^2|M^{0\nu}(A,Z)|}. \tag{11}$$

In this work we consider the RQRPA $0\nu\beta\beta$-decay matrix elements of Ref. [38], which were determined with the help of the average values of the measured $2\nu\beta\beta$-decay half-lives. They are given in Table 1 of Ref. [20]. In the case of ^{136}Xe for which $2\nu\beta\beta$ decay has not been observed yet, the current lower limit on the half-life is considered as a reference. For the ^{130}Te isotope we took into account the recent measurement of the $2\nu\beta\beta$-decay half-life of ^{130}Te by the CUORE Collaboration: $T^{0\nu\, expt}_{1/2}=[6.1\pm1.4\ (\mathrm{stat})+2.9-3.5\ (\mathrm{sys})]\times10^{20}$ yr [71]. This value is smaller by about a factor of 3 than the previously considered average value given in Ref. [20]. However, this has only a small impact on the calculated $0\nu\beta\beta$-decay matrix element, which increases by about 20%.

In Table I we present both the $0\nu\beta\beta$-decay [38] and the $2\nu\beta\beta$-decay matrix elements, the ratio $\mathcal{R}^{2\nu/0\nu}(A,Z)$, the average and measured $2\nu\beta\beta$-decay half-lives, and the current experimental limits on the $0\nu\beta\beta$-decay half-life and the half-lifetimes of the $0\nu\beta\beta$-decay, which are expected in future experiments after 5 yr of data taking [20]. By glancing at Table I we see that the values of $\mathcal{R}^{2\nu/0\nu}$ for various nuclei differ significantly from each other. This is connected with the fact that the $2\nu\beta\beta$-decay matrix element is sensitive to the energy distribution of the β strengths via the energy denominator. The largest value is associated with the $A=100$ system for which the ground state of the intermediate nucleus is the 1^+ state.

The current upper limits on the effective Majorana neutrino mass $m_{\beta\beta}$ and the expected sensitivities of running and planned experiments to this parameter for $A=76$, 100, 130, and 136 are listed in Table I. We see that the Heidelberg-Moscow experiment [37] offers the most restrictive limit $|m_{\beta\beta}| \leqslant 0.55$ eV. In future the sensitivity to $|m_{\beta\beta}|$ might be increased by about one order of magnitude (see Table I).

If the $0\nu\beta\beta$ decay is observed, the question of the uncertainty in the deduced value on $|m_{\beta\beta}|$ will be a subject of great importance. This problem can be solved by observation of the $0\nu\beta\beta$ decay of several nuclei. Any uncertainty in the nuclear matrix element reflects directly on measurements of $|m_{\beta\beta}|$. The spread of the $|m_{\beta\beta}|$ values associated with different nuclei will allow conclusions about the accuracy of the calculated $0\nu\beta\beta$-decay matrix elements. Another scenario was proposed in Ref. [72]. It was suggested to study the ratios of $0\nu\beta\beta$-decay matrix elements of different nuclei deduced from the corresponding half-lives. Unfortunately, the uncertainty of the absolute value of the $0\nu\beta\beta$-decay matrix elements cannot be established in this way. The first results obtained within the recently improved QRPA procedure for calculating nuclear matrix element [38] are encouraging and suggest that the uncertainty for a given isotope is of the order of tenths of percent. It goes without saying that this has to be confirmed by further theoretical analysis.

III. THE EFFECTIVE MAJORANA NEUTRINO MASS AND NEUTRINO OSCILLATION DATA

The effective Majorana mass is determined from the absolute values of neutrino masses m_i and the elements of the first row of the neutrino mixing matrix U_{ei} $(i=1,2,3)$. Taking into account existing neutrino oscillation data, we will discuss now a possible value of $|m_{\beta\beta}|$.

In the Majorana case all the elements U_{ek} are complex quantities,

$$U_{ek}=|U_{ek}|e^{i\alpha_k}, \tag{12}$$

where α_k is the Majorana *CP* phase. If *CP* invariance in the lepton sector holds, we have

$$U_{ek}=U^*_{ek}\eta_k, \tag{13}$$

where $\eta_k=i\rho_k$ $(\rho_k=\pm1)$ is the *CP* parity of the neutrino with a definite mass. Thus, in the case of *CP* invariance we have

$$2\alpha_k=\frac{\pi}{2}\rho_k. \tag{14}$$

The neutrino oscillation data are compatible with two types of neutrino mass spectra:[4]

(1) "The normal" mass spectrum $m_1<m_2<m_3$,

$$\Delta m^2_{21}\simeq\Delta m^2_{\rm sol}, \quad \Delta m^2_{32}\simeq\Delta m^2_{\rm atm}.$$

(2) "The inverted" mass spectrum $m_3<m_1<m_2$,

$$\Delta m^2_{21}\simeq\Delta m^2_{\rm sol}, \quad \Delta m^2_{31}\simeq-\Delta m^2_{\rm atm}.$$

[4] Δm^2_{ik} is defined as follows: $\Delta m^2_{ik}=m^2_i-m^2_k$.

For the neutrino masses in the case of the normal spectrum we have

$$m_2\simeq\sqrt{m_1^2+\Delta m^2_{\rm sol}}, \quad m_3\simeq\sqrt{m_1^2+\Delta m^2_{\rm atm}}, \tag{15}$$

where we took into account that $\Delta m^2_{\rm sol}\ll\Delta m^2_{\rm atm}$. In the case of the inverted spectrum we have

$$m_2\simeq m_1\simeq\sqrt{m_3^2+\Delta m^2_{\rm atm}}. \tag{16}$$

The elements $|U_{ei}|^2$ for both types of neutrino mass spectra are given by

$$|U_{e1}|^2=\cos^2\theta_{13}\cos^2\theta_{12},$$
$$|U_{e2}|^2=\cos^2\theta_{13}\sin^2\theta_{12}, \quad |U_{e3}|^2=\sin^2\theta_{13}. \tag{17}$$

The mixing angle θ_{12} was determined from the data of the solar neutrino experiments and the KamLAND reactor experiment. From the latest analysis of the existing data for the best-fit value of $\sin^2\theta_{12}$ it was found that [2]

$$\sin^2\theta_{12}\simeq\sin^2\theta_{\rm sol}=0.29. \tag{18}$$

For the angle θ_{13} only the upper bound is known. From the exclusion plot obtained from the data of the reactor experiment CHOOZ [75] at $\Delta m^2_{32}=2\times10^{-3}$ eV^2 (the Super-Kamiokande best-fit value) we have

$$\sin^2\theta_{13}\leqslant5\times10^{-2}. \tag{19}$$

For the minimal neutrino mass m_1 (m_3) we also know only an upper bound. From the data of the tritium Mainz [76] and Troitsk [77] experiments,

$$m_1\leqslant2.2 \ \text{eV}. \tag{20}$$

In the future tritium experiment KATRIN [78], the sensitivity $m_1\simeq0.25$ eV is planned to be achieved.

Important information about the sum of the neutrino masses can be obtained from cosmological data. From the Wilkinson Microwave Anisotropy Probe (WMAP) and 2° Field Galaxy Redshift Survey data [79],

$$\sum_i m_i\leqslant0.7 \ \text{eV}. \tag{21}$$

More conservative bound was obtained in [80] from analysis of the latest Sloan Digital Sky Survey data and WMAP data. The best-fit value of $\Sigma_i m_i$ was found to be equal to zero. For the upper bound one obtains

$$\sum_i m_i\leqslant1.7 \ \text{eV}. \tag{22}$$

For the case of three massive neutrinos this bound implies

$$m_1\leqslant0.6 \ \text{eV}. \tag{23}$$

The value of $|m_{\beta\beta}|$ depends on the neutrino mass spectrum [27–36]. We discuss three "standard" neutrino mass spectra, which are frequently considered in the literature.

(1) *The normal hierarchy of neutrino masses*,[5] which corresponds to the case

$$m_1 \ll m_2 \ll m_3 . \tag{24}$$

In this case neutrino masses are known from neutrino oscillation data. We have

$$m_1 \ll \sqrt{\Delta m^2_{\rm sol}}, \quad m_2 \simeq \sqrt{\Delta m^2_{\rm sol}}, \quad m_3 \simeq \sqrt{\Delta m^2_{\rm atm}}. \tag{25}$$

For the effective Majorana mass we have the following upper and lower bounds:

$$|m_{\beta\beta}| \leq (\cos^2\theta_{13}\sin^2\theta_{\rm sol}\sqrt{\Delta m^2_{\rm sol}} + \sin^2\theta_{13}\sqrt{\Delta m^2_{\rm atm}}) \tag{26}$$

and

$$|m_{\beta\beta}| \geq |\cos^2\theta_{13}\sin^2\theta_{\rm sol}\sqrt{\Delta m^2_{\rm sol}} - \sin^2\theta_{13}\sqrt{\Delta m^2_{\rm atm}}|. \tag{27}$$

Using the best-fit values of the solar neutrino oscillation parameters [see Eqs. (2) and (18)] and the upper bound (19) we end up with

$$\cos^2\theta_{13}\sin^2\theta_{\rm sol}\sqrt{\Delta m^2_{\rm sol}} \simeq 2.32\times 10^{-3}\ \ \text{eV},$$
$$\sin^2\theta_{13}\sqrt{\Delta m^2_{\rm atm}} \leq 2.24\times 10^{-3}\ \ \text{eV}. \tag{28}$$

We note that the first and second terms on the right hand side of Eq. (27) differ only slightly from each other. This means that the value of the effective Majorana neutrino mass $m_{\beta\beta}$ might be close to zero. For the choice of three possible values $\sin^2\theta_{13} = 0.05$, 0.01, and 0.00 we end up with allowed intervals for $|m_{\beta\beta}|$:

$$\sin^2\theta_{13} = \begin{cases} 0.05 \Rightarrow 8.5\times 10^{-5}\ \text{eV} \leq |m_{\beta\beta}| \leq 4.6\times 10^{-3}\ \text{eV}, \\ 0.01 \Rightarrow 2.0\times 10^{-3}\ \text{eV} \leq |m_{\beta\beta}| \leq 2.9\times 10^{-3}\ \text{eV}, \\ 0.00 \Rightarrow |m_{\beta\beta}| = 2.4\times 10^{-3}\ \text{eV}. \end{cases} \tag{29}$$

From Eq. (29) it follows that a smaller value of $\sin^2\theta_{13}$ implies a narrower range of the allowed values of $m_{\beta\beta}$. We also conclude that in the case of the normal neutrino mass hierarchy the upper bound $|m_{\beta\beta}| \leq 4.6\times 10^{-3}$ eV is far from the value which can be reached in the $0\nu\beta\beta$-decay experiments of the next generation.

(2) *Inverted hierarchy of neutrino masses*. It is given by the condition

$$m_3 \ll m_1 < m_2 . \tag{30}$$

In this case for the neutrino masses we have

$$m_3 \ll \sqrt{\Delta m^2_{\rm atm}}, \quad m_1 \simeq \sqrt{\Delta m^2_{\rm atm}},$$

$$m_2 \simeq \sqrt{\Delta m^2_{\rm atm}}\left(1 + \frac{\Delta m^2_{\rm sol}}{2\Delta m^2_{\rm atm}}\right) \simeq \sqrt{\Delta m^2_{\rm atm}}. \tag{31}$$

The effective Majorana mass is given by

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\rm atm}}\left|\sum_{i=1,2} U^2_{ei}\right|. \tag{32}$$

Neglecting small ($\leq 5\%$) corrections due to $|U_{e3}|^2$, for $|m_{\beta\beta}|$ we obtain

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\rm atm}}(1 - \sin^2 2\theta_{\rm sol}\sin^2\alpha_{21})^{1/2}, \tag{33}$$

where $\alpha_{21} = \alpha_2 - \alpha_1$ is the Majorana *CP*-phase difference.

Thus, in the case of the inverted mass hierarchy the value of the effective Majorana mass can lie in the range

$$\cos 2\theta_{\rm sol}\sqrt{\Delta m^2_{\rm atm}} \leq |m_{\beta\beta}| \leq \sqrt{\Delta m^2_{\rm atm}}. \tag{34}$$

The bounds in Eq. (34) correspond to the case of *CP* conservation: the upper bound corresponds to the case of equal *CP* parities of ν_2 and ν_3 and the lower bound to the case of opposite *CP* parities. From Eqs. (18) and (34) we get

$$0.42\sqrt{\Delta m^2_{\rm atm}} \leq |m_{\beta\beta}| \leq \sqrt{\Delta m^2_{\rm atm}}. \tag{35}$$

Let us assume that the problem of nuclear matrix elements will be solved (say, in the manner we discussed before). If the measured value of $|m_{\beta\beta}|$ is within the range given in Eq. (35), it will be an indication in favor of the inverted hierarchy of neutrino masses.[6] The only unknown parameter that enters into the expression for the effective Majorana mass in the case of the inverted hierarchy is $\sin^2\alpha_{21}$. Thus, the measurement of $|m_{\beta\beta}|$ might allow, in principle, information to be obtained about the Majorana *CP* phase difference $|\alpha_{21}|$ [28,29]. It would require, however, a precise measurement of the $0\nu\beta\beta$ half-time.

(3) *Almost degenerate neutrino mass spectrum.* In the two cases of neutrino mass spectra discussed above, the lightest neutrino mass was assumed to be small. The existing bounds on the absolute value of the neutrino mass [see Eqs. (20) and (23)] do not exclude the possibility that the lightest neutrino mass is much larger than $\sqrt{\Delta m^2_{\rm atm}}$. In this case we have

$$m_1 \simeq m_2 \simeq m_3 , \tag{36}$$

and the effective Majorana mass takes the form

[5] Notice that the masses of charged leptons and up and down quarks satisfy a hierarchy of the type (24).

[6] Notice that the type of neutrino mass spectra (normal or inverted) can be determined via the comparison of the probabilities of $\nu_\mu \to \nu_e$ and $\overline{\nu_\mu} \to \overline{\nu_e}$ transitions in long baseline neutrino experiments [81].

$$|m_{\beta\beta}| \simeq m_1 \left| \sum_{i=1}^{3} U_{ei}^2 \right|. \tag{37}$$

By neglecting the small contribution of the parameter $|U_{e3}|^2$, from Eq. (37) we get

$$|m_{\beta\beta}| \simeq m_1 (1 - \sin^2 2\theta_{\rm sol} \sin^2 \alpha_{21})^{1/2}. \tag{38}$$

Using the best-fit value (18) we obtain

$$0.42 m_1 \leqslant |m_{\beta\beta}| \leqslant m_1. \tag{39}$$

Thus, if it occurs that the effective Majorana mass $|m_{\beta\beta}|$ is relatively large (much larger than $\sqrt{\Delta m^2_{\rm atm}} \simeq 4.5 \times 10^{-2}$ eV), it signifies that the neutrino mass spectrum is almost degenerate. If the case $|m_{\beta\beta}| \gg \sqrt{\Delta m^2_{\rm atm}} \simeq 4.5 \times 10^{-2}$ eV is confirmed by $0\nu\beta\beta$-decay experiments, the explanation could be a degenerate neutrino mass spectrum. From Eq. (39) for the common neutrino mass we get the range

$$|m_{\beta\beta}| \leqslant m_1 \leqslant 2.38 |m_{\beta\beta}|. \tag{40}$$

From Eq. (38) it is obvious that if the common mass m_1 is determined from β-decay measurements and/or cosmological data, the evidence of the $0\nu\beta\beta$ decay will allow valuable information to be deduced about the Majorana *CP* phase difference via the accurate measurement of the $0\nu\beta\beta$ half-time.

For the purpose of illustration of the problem of the neutrino mass hierarchy we will assume that $\ll$ and $\gg$ in Eqs. (25) and (31) can be represented by a factor of 5. Then we have

Normal hierarchy (NH):

$$m_1 \ll \sqrt{\Delta m^2_{\rm sol}},$$

$$m_1 \leqslant \sqrt{\Delta m^2_{\rm sol}}/5 = 1.7 \times 10^{-3} \ \ \text{eV};$$

Inverted hierarchy (IH):

$$m_3 \ll \sqrt{\Delta m^2_{\rm atm}},$$

$$m_3 \leqslant \sqrt{\Delta m^2_{\rm atm}}/5 = 8.9 \times 10^{-3} \ \ \text{eV};$$

Almost degenerate (AD):

$$m_1, m_3 \gg \sqrt{\Delta m^2_{\rm atm}},$$

$$m_1, m_3 \geqslant 5 \sqrt{\Delta m^2_{\rm atm}} = 0.22 \ \ \text{eV}. \tag{41}$$

It is worthwhile to notice that in the case of the almost degenerate neutrino mass spectrum there is an upper limit from the cosmological data [see Eq. (23)]: $m_1, m_3 \leqslant 0.6$ eV. The bounds in Eq. (41) are displayed in Fig. 1.

In Table II we give the values of the neutrino masses m_1, m_2, and m_3, and the minimal and maximal predicted values of $|m_{\beta\beta}|$ in the cases of the normal and inverted hierarchies

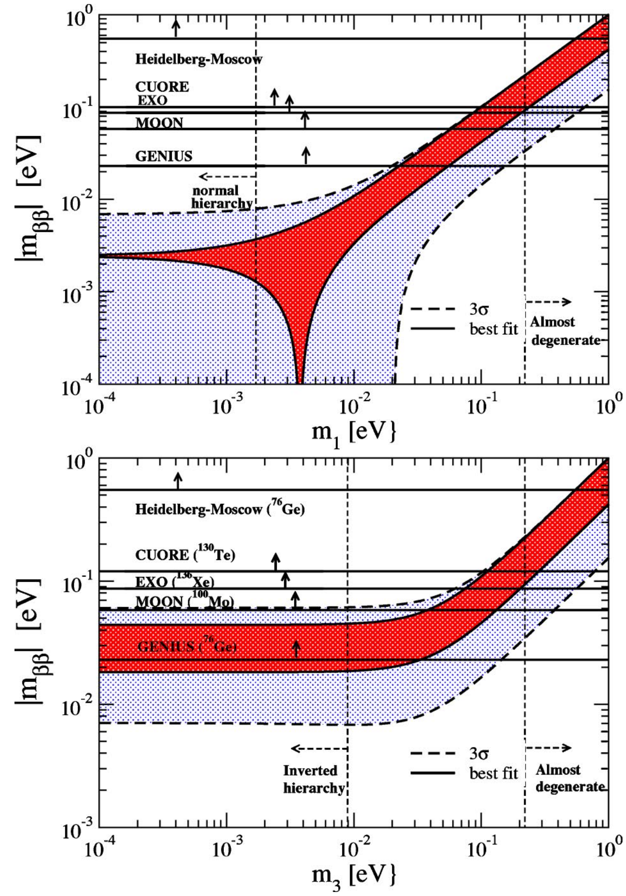

FIG. 1. The effective Majorana neutrino mass $m_{\beta\beta}$ as a function of the lightest neutrino mass m_1 (the normal hierarchy of neutrino masses, upper panel) and m_3 (the inverted hierarchy of neutrino masses, lower panel). The ranges of the normal hierarchy ($m_1 \leqslant 1.7 \times 10^{-3}$ eV) and inverted hierarchies ($m_3 \leqslant 8.9 \times 10^{-3}$ eV) of neutrino masses and the almost degenerate ($m_1, m_3 \geqslant 0.22$ eV) neutrino mass spectrum [see Eq. (41) and the text above it for definition] are indicated by dashed arrows. The best-fit results (the region with solid line boundaries) correspond to the parameter set $\Delta m^2_{\rm sol} = 7.1 \times 10^{-5}$ eV^2, $\Delta m^2_{\rm atm} = 2.0 \times 10^{-3}$ eV^2, $\sin^2\theta_{12} = 0.29$ [2,1,13], and $\sin^2\theta_{13} = 0.00$. The 3σ results (the region with dashed line boundaries) correspond to the global fit of Ref. [82]. The sensitivities of future experiments on the search for the $0\nu\beta\beta$ decay of different isotopes are indicated with horizontal solid bold lines.

of the neutrino masses and the almost degenerate neutrino mass spectrum. Three values for $\sin^2\theta_{13}$ compatible with the CHOOZ upper bound are considered: $\sin^2\theta_{13} = 0.00$, 0.01, and 0.05. We see that by decreasing $\sin^2\theta_{13}$ the allowed interval for $|m_{\beta\beta}|$ becomes narrower. This behavior is apparent especially in the case of the normal hierarchy of neutrino masses. For this scenario of the neutrino mass spectrum the largest value of $|m_{\beta\beta}|$ is of the order of 5×10^{-3} eV. None of the planned $0\nu\beta\beta$-decay experiments can reach such a level of sensitivity to $|m_{\beta\beta}|$ (see Table I). In the case of the inverted hierarchy, $|m_{\beta\beta}|$ depends only weakly on the angle θ_{13}, and its maximal value is about an order of magnitude larger than in the case of the normal hierarchy. This sensitivity can be reached only by the future Ge $0\nu\beta\beta$-decay experiments (see Fig. 1). For this type of neutrino mass spec-

TABLE II. The effective Majorana neutrino mass $|m_{\beta\beta}|$ in the cases of the normal and inverted hierarchies of neutrino masses and the almost degenerate neutrino mass spectrum [see Eq. (41) and the text above]. The best-fit values $\Delta m^2_{\rm sol}=7.1\times10^{-5}$ eV^2, $\Delta m^2_{\rm atm}=2.0\times10^{-3}$ eV^2, and $\sin^2\theta_{12}=0.29$ are considered [2,1,13]. The results are presented for three values of the angle θ_{13} from the CHOOZ allowed range $\sin^2\theta_{13}\leqslant 0.05$ [75].

Normal hierarchy of ν masses: $m_1 \ll m_2 \ll m_3$				
m_1 (10^{-3} eV)	m_2 (10^{-3} eV)	m_3 (10^{-2} eV)	$\sin^2\theta_{13}$	$\lvert m_{\beta\beta}\rvert$ (10^{-3} eV)
(0, 1.7)	(8.43, 8.60)	(4.47, 4.48)	0.00	(1.29, 3.70)
			0.01	(0.83, 4.11)
			0.05	(0.00, 5.75)
Inverted hierarchy of ν masses: $m_3 \ll m_1 < m_2$				
m_3 (10^{-3} eV)	m_1 (10^{-2} eV)	m_2 (10^{-2} eV)	$\sin^2\theta_{13}$	$\lvert m_{\beta\beta}\rvert$ (10^{-2} eV)
(0, 8.9)	(4.39, 4.48)	(4.47, 4.56)	0.00	(1.82, 4.50)
			0.01	(1.80, 4.47)
			0.05	(1.72, 4.32)
Almost degenerate ν mass spectrum: $m_1 \simeq m_2 \simeq m_3$				
m_1 (eV)	m_2 (eV)	m_3 (eV)	$\sin^2\theta_{13}$	$\lvert m_{\beta\beta}\rvert$ (eV)
(0.22, 0.60)	(0.22, 0.60)	(0.22, 0.60)	0.00	(0.092, 0.60)
			0.01	(0.089, 0.60)
			0.05	(0.077, 0.60)

trum the maximal and minimal allowed values of $|m_{\beta\beta}|$ differ by about a factor of 2.5. Thus, it will be possible to draw conclusions about the Majorana *CP* phase difference in the case of observation of $0\nu\beta\beta$ decay with $|m_{\beta\beta}|$ in the range $(1.8–4.5)\times10^{-2}$ eV, if the uncertainties of the nuclear matrix elements are small. We stress that in order to find some information about the *CP* phase difference the value of the lightest neutrino must be known with good enough precision. Tables I and II suggest[7] that the non-Ge experiments NEMO3, MOON (^{100}Mo), CUORE (^{130}Te), XMASS, and EXO (^{136}Xe) will be able to test mainly the case of the almost degenerate mass spectrum.

There is a very good potential for discovery of the $0\nu\beta\beta$ decay in the GEM, GENIUS, and Majorana experiments, which plan to use an enriched ^{76}Ge source. In Fig. 2 the half-life of the $0\nu\beta\beta$ decay of ^{76}Ge is plotted as a function of the lightest neutrino mass. We see that these three experiments might observe the $0\nu\beta\beta$ decay in the cases of an almost degenerate spectrum and an inverted hierarchy of neutrino masses. Let us stress that it is very important to achieve high sensitivity in several other experiments also, using other nuclei as the radioactive source. This will allow important information to be obtained about the accuracy of the nuclear matrix elements involved and the effect of the *CP* Majorana phases to be discussed. The expected half-lifetimes of $0\nu\beta\beta$ decay of ^{100}Mo, ^{130}Te, and ^{136}Xe calculated with the nuclear matrix elements of Ref. [38] with minimal neutrino mass considered as a parameter are shown in Figs. 3, 4, and 5.

[7]Let us stress that the values of the effective Majorana mass $|m_{\beta\beta}|$, given in Tables I and II were obtained with the nuclear matrix elements of Ref. [21].

IV. SUMMARY AND CONCLUSIONS

After the discovery of neutrino oscillations in the atmospheric, solar, and reactor KamLAND experiments, the problem of the nature of neutrinos with definite masses (Dirac or Majorana?) has become very important. The most sensitive process to possible violation of the lepton number and small Majorana neutrino masses is the neutrinoless double β decay. At present many new experiments searching for the $0\nu\beta\beta$ decay of ^{76}Ge, ^{100}Mo, ^{130}Te, ^{136}Xe, and other nuclei are in preparation or under consideration. In these experiments, about an order of magnitude improvement of the sensitivity to the effective Majorana mass $|m_{\beta\beta}|$ in comparison with the current Heidelberg-Moscow [37] and IGEX [42] experiments is expected. If the $0\nu\beta\beta$ decay is observed, it will allow one not only to establish that massive neutrinos are Majorana particles but also to reveal the character of the neutrino mass spectrum and the absolute scale of the neutrino masses.

The data from neutrino oscillation experiments allow ranges of possible values of the effective Majorana mass for different neutrino mass spectra to be predicted. Thus, in order to discriminate different possibilities, it is important not only to observe the $0\nu\beta\beta$ decay but also to *measure* the effective Majorana mass $|m_{\beta\beta}|$.

From the measured half-lifetime of the $0\nu\beta\beta$ decay only the product of the effective Majorana mass and the nuclear matrix element can be determined. There is a widespread opinion that the current uncertainty in the $0\nu\beta\beta$-decay matrix elements is of the order of a factor of 3 and more [83]. Let us stress that a very important source of the uncertainty is associated with the fixing of the nuclear structure parameter space. Recently, surprising results were obtained by fixing of the particle-particle interaction strength to the

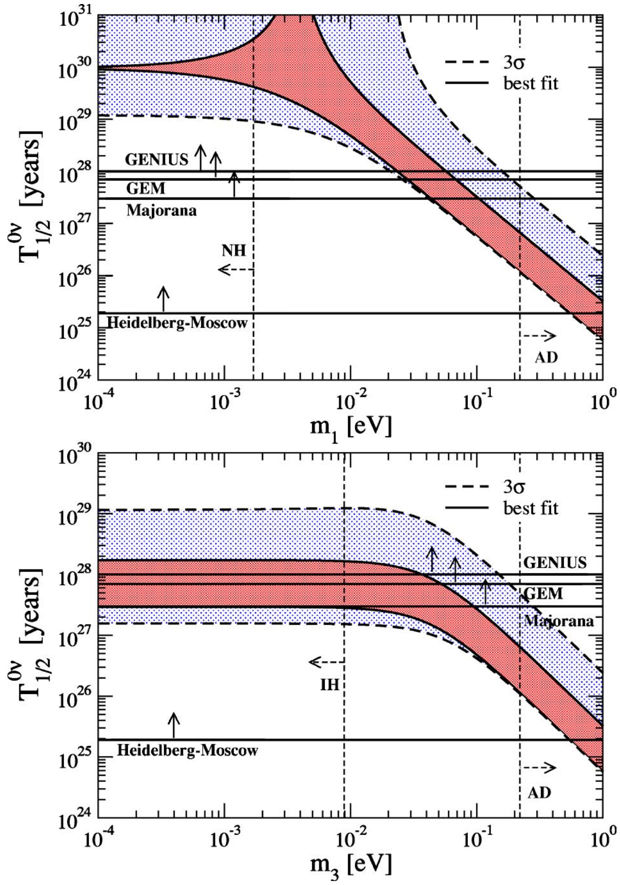

FIG. 2. The neutrinoless double beta half-life of ^{76}Ge as a function of the lightest neutrino mass m_1 (upper panel) and m_3 (lower panel). Conventions are the same as in Fig. 1. We see that all three planned Ge experiments Majorana, GEM, and GENIUS can check the neutrino mixing scenario of the inverted hierarchy of masses. NH, IH, and AD denote the normal hierarchy, and the inverted hierarchy of neutrino masses and the almost degenerate neutrino mass spectrum, respectively.

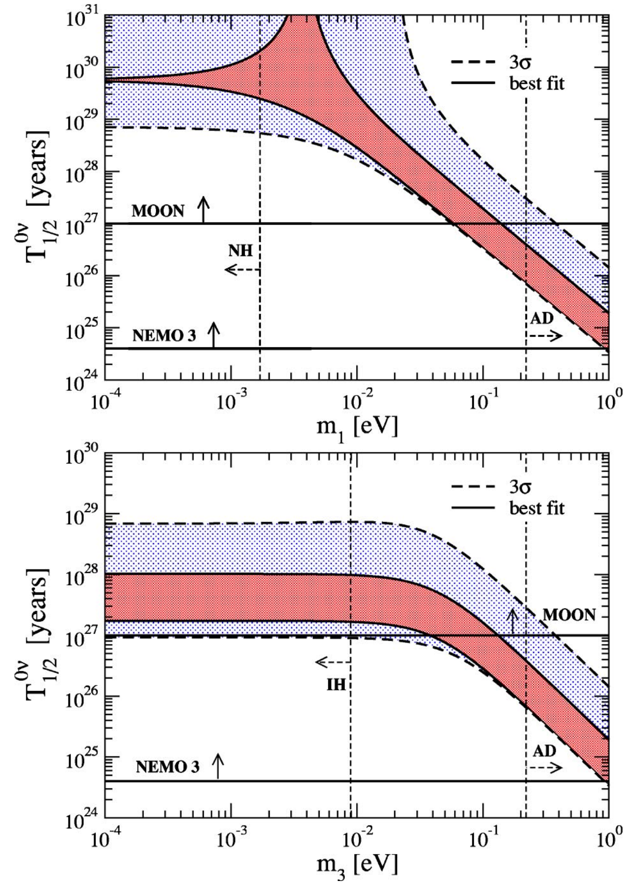

FIG. 3. The neutrinoless double beta half-life of ^{100}Mo as a function of the lightest neutrino mass m_1 (upper panel) and m_3 (lower panel). Conventions are the same as in Fig. 2.

$2\nu\beta\beta$-decay rate [38]. This procedure allowed reduction of the theoretical uncertainty of the $0\nu\beta\beta$-decay matrix elements for ^{76}Ge, ^{100}Mo, ^{130}Te, and ^{136}Xe within the QRPA. It will be important to confirm this result for other double beta decaying isotopes and for various QRPA extensions. There is also the possibility of building a single QRPA theory with all the studied implementations. Improvement of the calculations of the nuclear matrix elements is a real theoretical challenge. There is a chance that the uncertainty of the calculated $0\nu\beta\beta$-decay matrix elements will be reduced down to the order of tenths of a percent. A possible test of the calculated nuclear matrix elements will be offered by observation of the $0\nu\beta\beta$ decay of several nuclei. The spread of the values of $|m_{\beta\beta}|$ associated with different isotopes will allow conclusions to be drawn about the quality of the nuclear structure calculations.

In this paper we considered $0\nu\beta\beta$-decay matrix elements with a reduced theoretical uncertainty [38] and determined the sensitivities of running and planned $0\nu\beta\beta$-decay experiments to the effective Majorana neutrino mass $|m_{\beta\beta}|$ for ^{76}Ge, ^{100}Mo, ^{130}Te, and ^{136}Xe. The effective Majorana neutrino mass was evaluated also by taking into account the results of the neutrino oscillation experiments. Three different cases of neutrino mass spectra were analyzed: (i) a normal hierarchical, (ii) an inverted hierarchical, and (iii) an almost degenerate mass spectrum. The best-fit values for $\Delta m^2_{\rm sol}$, $\Delta m^2_{\rm atm}$, and $\sin^2\theta_{12}$ were considered. The analysis was performed for three values of the parameter $\sin^2\theta_{13}$ ($\sin^2\theta_{13}=0.00,0.01,0.05$). A selected group of future experiments associated with the above isotopes was discussed. It was found that the NEMO3, MOON, CUORE, XMASS, and EXO experiments have the possibility of confirming or ruling out the possibility of the almost degenerate neutrino mass spectrum. The planned Ge experiments (Majorana, GEM, and GENIUS) seem to have a very good sensitivity to $|m_{\beta\beta}|$. These experiments will observe the $0\nu\beta\beta$ decay, if the neutrinos are Majorana particles and there is an inverted hierarchy of neutrino masses.

Finally, by taking into account existing values of the neutrino oscillation parameters we present some general conclusions.

If the $0\nu\beta\beta$ decay is not observed in the experiments of the next generation and

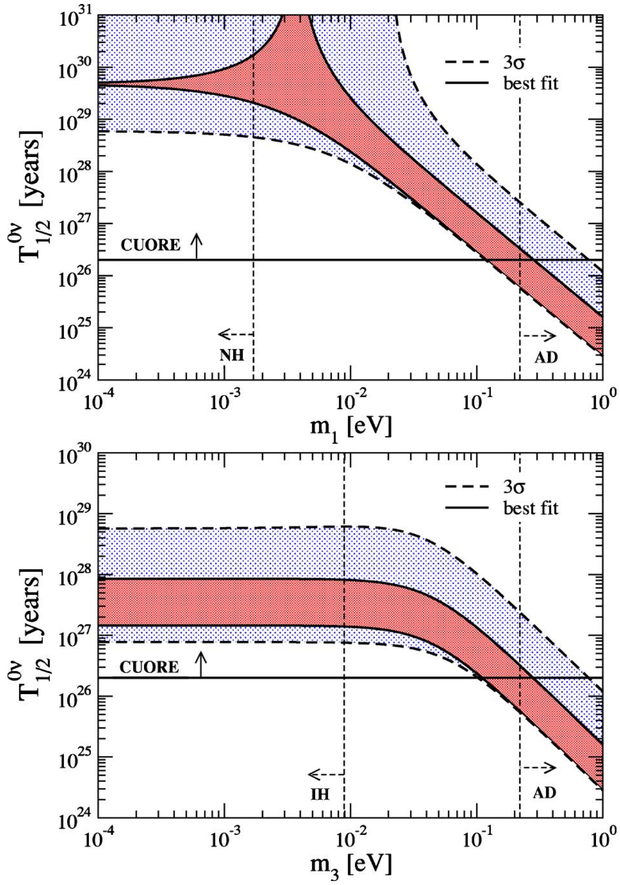

FIG. 4. The neutrinoless double beta half-life of ^{130}Te as a function of the lightest neutrino mass m_1 (upper panel) and m_3 (lower panel). Conventions are the same as in Fig. 2.

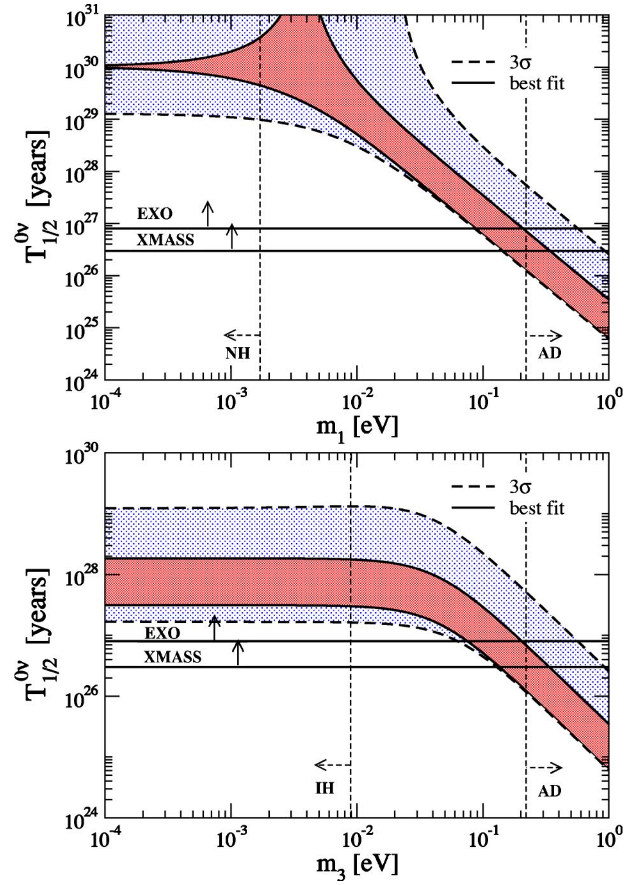

FIG. 5. The neutrinoless double beta half-life of ^{136}Xe as a function of the lightest neutrino mass m_1 (upper panel) and m_3 (lower panel). Conventions are the same as in Fig. 2.

$$|m_{\beta\beta}| \leq \text{ a few } 10^{-2} \text{ eV},$$

either massive neutrinos are Dirac particles or massive neutrinos are Majorana particles and a normal neutrino mass hierarchy is realized in nature. The observation of $0\nu\beta\beta$ decay with

$$|m_{\beta\beta}| \geq 4.5 \times 10^{-2} \text{ eV}$$

will exclude the normal hierarchy of neutrino masses.

If the $0\nu\beta\beta$ decay is observed and

$$0.42\sqrt{\Delta m^2_{\text{atm}}} \leq |m_{\beta\beta}| \leq \sqrt{\Delta m^2_{\text{atm}}},$$

it will be an indication in favor of the inverted hierarchy of neutrino masses.

If the $0\nu\beta\beta$ decay is observed in future experiments and

$$|m_{\beta\beta}| \geq \sqrt{\Delta m^2_{\text{atm}}},$$

the neutrino mass spectrum is almost degenerate and a range for the common neutrino mass can be determined.

If from the future tritium neutrino experiments or from future cosmological measurements the common neutrino mass is determined, it will be possible to predict the value of the effective Majorana neutrino mass:

$$0.42 m_1 \leq |m_{\beta\beta}| \leq m_1.$$

Nonobservation of $0\nu\beta\beta$ decay with the effective Majorana mass $|m_{\beta\beta}|$ in this range will mean that the neutrinos are Dirac particles (or other mechanisms of violation of the lepton number are involved).

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